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Mirror symmetry and toric degenerations of partial flag manifolds.

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This paper proposes a construction for mirror manifolds to Calabi-Yau complete intersections in partial flag manifolds, generalising the ideas in the authors' earlier paper [Nuclear Phys. B **514** (1998), no. 3, 640–666; [MR1619529 \(99m:14074\)](#)] which applied to Grassmannians. The main component of this proposal is a combinatorial construction which gives a degeneration of any partial flag manifold to a toric variety. Thus a complete intersection in the flag manifold degenerates to a complete intersection in the corresponding toric variety. Now one knows what the mirror to a complete intersection Calabi-Yau in a toric variety should be by Borisov's construction, and then one hopes to find the mirror family to the original Calabi-Yau as a subfamily of this mirror family. The choice of this subfamily is guided by a structure arising out of A. Givental's study of the quantum cohomology of complete flag manifolds [in *Topics in singularity theory*, 103–115, Amer. Math. Soc. Transl. Ser. 2, 180, Amer. Math. Soc., Providence, RI, 1997; [MR1767115 \(2001d:14063\)](#)].

Reviewed by [Mark Gross](#)

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